MATH 464 (THEORY OF PROBABILITY) HOMEWORK 6

FALL 2017

<u>Due on:</u> Tuesday 10-03-2017.

(1) Suppose $X_1, \ldots, X_n$ are independent random variables with $\mathbb{E}(X_i) = \mu$ and $\text{Var}(X_i) = \sigma$ for all $i = 1, \ldots, n$, Find $\mathbb{E}((X_1 + \ldots + X_n)^2)$.

(2) Let X and Y be independent discrete random variables. Given that

$$\mathbb{E}(X^n) = 2^{n-1}, \text{ and } \mathbb{E}(Y^n) = \begin{cases} 0, & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases} \quad \text{for } n = 1, 2, \ldots$$

(a) Give an example of such random variables X and Y.
(b) Let $Z = 2X + Y$. Find the mean and variance of Z.
(c) Let $W = Y^2 - 2YX^2$. Find the mean and variance of W.

(3) Let X and Y be independent geometric random variables with means $\mathbb{E}(X) = 2$ and $\mathbb{E}(Y) = 3$.
(a) Find the joint probability mass function of X and Y.
(b) Find $\mathbb{P}(X + Y \leq 3)$.
(c) Consider $W = \min\{X, Y\}$ and $Z = \max\{X, Y\}$. Find the joint probability mass function of W and Z.

(4) Suppose that U and V are two independent random variables, each take the values of -1 and 1 only, and $\mathbb{P}(U = 1) = \mathbb{P}(V = 1) = 1/2$. A third random variable W is defined by $W = UV$. Show that the random variables U, V, and W are pairwise-independent but they are not independent.